COMMENTS ON "MEASURING THE GRAVITY SPEED BY VLBI"

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Einstein gravity with extra dimensions or alternative gravity theories might suggest that the gravity propagation speed can be different from the light speed. Such a difference may play a vital role in the primordial universe. In recent, Kopeikin and Fomalont claimed the first measurement of the gravity speed by VLBI. However, the measurement has no relevance with the speed of gravity as I had shown before the observation was done. It seems that our conclusion has been established well by re-examining recent papers with great care.

1 Introduction

Einstein gravity with extra dimensions or alternative gravity theories might suggest that the gravity propagation speed can be different from the light speed. The Shapiro time delay plays an important role in experimental verification of Einstein's theory of general relativity. In fact, VLBI (very long baseline interferometry) confirms the validity of general relativity. The accuracy will be achieved within a few picoseconds (ps), namely about 10 microarcseconds (μas), for instance by VERA (VLBI Exploration of Radio Astrometry [1]). This is why the correction to the Shapiro time delay has been intensively investigated [2,3]. Kopeikin found that the excess time delay caused by Jupiter can be measured on the 8th of September 2002. He also argued that the excess was due to the propagation of gravity, which could be tested through the observation. Indeed, Kopeikin and Fomalont made the observation and claimed the first measurement of the *gravity* speed [4]. Before the measurement, however, it had been shown that it comes from the propagation of light but not gravity [5].

The primary reason against his conclusion is based on the post-Newtonian approximation of general relativity: In the approximation, we perform expansions in the inverse of light velocity c, by considering all quantities are perturbations around the Newtonian parts. The deflection of light, the perihelion shift of Mercury and the time delay occur at the first post-Newtonian order $O(c^{-2})$. Actually, using these three effects, the classical tests have been done to confirm the validity of general relativity (For a thorough review, see Will[6]). The propagation of gravity appears at $O(c^{-4})$ because mass dipole moments vanish in contrast to electromagnetism [7,6]. The effect of the radiation reaction of quadrupole gravitational waves [8,7,6] has been confirmed through the observation of decaying orbital period of Hulse-Taylor binary pulsar[9]. On the other hand, the Kopeikin's excess time delay for the standard Shapiro delay at $O(c^{-2})$ is $O(v_J/c^3)$, where v_J is the velocity of Jupiter. The order of the excess is lower than that of the propagation of gravity, so that the excess cannot be caused by the gravity propagation. The origin of the excess is clarified in the following.

2 Shapiro delay in retarded time

Since a massive body produces gravitational fields as a curved spacetime, a light signal will take a longer time to traverse a given spatial distance than it would if Newtonian theory were valid. In deriving the Shapiro time delay, the Einstein equation for the gravitational field and the null geodesics for the light ray are solved up to $O(c^{-2})$. In particular, the Einstein equation is reduced to Poisson-type equations, so that the propagation of gravity is not incorporated. The Shapiro delay for a light signal from an emitter to an observer is obtained in a logarithmic form [10,6].

Let us consider a baseline denoted by $\boldsymbol{B}$; at t_1 and t_2, the light signals from a quasar reach the first and second stations which locate at $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$, respectively, so that we can define the baseline as the spatial interval between the simultaneous events $\boldsymbol{B} = \boldsymbol{x}_1(t_1) - \boldsymbol{x}_2(t_1)$. Each station is denoted by i later. Since the Shapiro delay is a consequence of integration of the null geodesics on the light cones, it is convenient and crucial to use s_1 and s_2, retarded time which is constant on each light cone emanating from events $(t_1, \boldsymbol{x}_1(t_1))$ and $(t_2, \boldsymbol{x}_2(t_2))$, so that we can have $\boldsymbol{x}_i(t_i) = \boldsymbol{x}_i(s_i)$ for $i = 1, 2$. Hence, the difference of the Shapiro delay between the baseline is expressed as

$$\Delta(t_1, t_2) = \frac{2GM}{c^3} \ln \frac{R_{1J} + \boldsymbol{K} \cdot \boldsymbol{R}_{1J}}{R_{2J} + \boldsymbol{K} \cdot \boldsymbol{R}_{2J}}, \tag{1}$$

where the unit vector from the Earth to the emitter of the light is denoted by $\boldsymbol{K}$, the position of Jupiter by $\boldsymbol{x}_J(t)$ and we defined $\boldsymbol{R}_{iJ} = \boldsymbol{x}_i(s_i) - \boldsymbol{x}_J(s_i)$ and $R_{iJ} = |\boldsymbol{R}_{iJ}|$ on each light cone labeled by $i = 1, 2$. Equation (1) is exact at the post-Newtonian order. In order to clarify the origin of the excess, we expand this equation approximately as follows.

Since the speed of Jupiter v_J is much *smaller* than c, we find

$$\boldsymbol{R}_{iJ} = \boldsymbol{r}_{iJ} + \frac{\boldsymbol{v}_J}{c} r_{iJ} + O(c^{-2}), \tag{2}$$

$$R_{iJ} = r_{iJ} + \frac{\boldsymbol{R}_{iJ} \cdot \boldsymbol{v}_J}{c} + O(c^{-2}), \tag{3}$$

where we used $\boldsymbol{x}_i(s_i) = \boldsymbol{x}_i(t_i)$ and denoted the spatial displacement vector between the simultaneous events by $\boldsymbol{r}_{iJ} = \boldsymbol{x}_i(t_i) - \boldsymbol{x}_J(t_i)$, and the interval by $r_{iJ} = |\boldsymbol{r}_{iJ}|$. Furthermore, since B is much *shorter* than $R \sim R_{iJ}$, we obtain

$$r_{2J} - r_{1J} = \boldsymbol{N}_{1J} \cdot \boldsymbol{B} + O\left(\frac{B^2}{R}\right), \tag{4}$$

where we defined $\boldsymbol{N}_{1J} = \boldsymbol{r}_{1J}/r_{1J}$. Hence, Eq. (1) becomes

$$\Delta(t_1, t_2) = \frac{2GM}{c^3} \Bigg(\ln \frac{r_{1J} + \boldsymbol{K} \cdot \boldsymbol{r}_{1J}}{r_{2J} + \boldsymbol{K} \cdot \boldsymbol{r}_{2J}} - \frac{\boldsymbol{B} \cdot \boldsymbol{v}_J + (\boldsymbol{N}_{1J} \cdot \boldsymbol{B})(\boldsymbol{K} \cdot \boldsymbol{v}_J)}{c(r_{1J} + \boldsymbol{K} \cdot \boldsymbol{r}_{1J})} + O(c^{-2}) \Bigg). \tag{5}$$

We denote by θ a *small* angle between the first station - source and the station - Jupiter at simultaneous time t. We obtain

$$\begin{aligned} \boldsymbol{N}_{1J} &= -\boldsymbol{K}\cos\theta + \boldsymbol{n}\sin\theta \\ &= -\left(1-\frac{\theta^2}{2}\right)\boldsymbol{K} + \theta\boldsymbol{n} + O(\theta^3), \end{aligned} \tag{6}$$

where $\boldsymbol{n}$ is a unit normal vector from the Jupiter to the light ray. Using this relation, we find

$$r_{1J} + \boldsymbol{K}\cdot\boldsymbol{r}_{1J} = \frac{\theta^2 r_{1J}}{2} + O(\theta^4), \tag{7}$$

$$\frac{r_{1J} + \boldsymbol{K}\cdot\boldsymbol{r}_{1J}}{r_{2J} + \boldsymbol{K}\cdot\boldsymbol{r}_{2J}} = 1 - \frac{2\boldsymbol{n}\cdot\boldsymbol{B}}{r_{1J}\theta} + O\left(\frac{B^2}{r^2}\right), \tag{8}$$

where we introduced $r \sim r_{1J} \sim r_{2J}$. Using these approximations, Eq. (5) is rewritten as

$$\begin{aligned} \Delta(t_1, t_2) = -\frac{4GM}{c^3} \Big(& \frac{\boldsymbol{n}\cdot\boldsymbol{B}}{r_{1J}\theta} \\ & + \frac{\boldsymbol{B}\cdot\boldsymbol{v}_J - (\boldsymbol{K}\cdot\boldsymbol{B})(\boldsymbol{K}\cdot\boldsymbol{v}_J)}{c r_{1J}\theta^2} + O(c^{-2}, c^{-1}B^2 r^{-2})\Big), \end{aligned} \tag{9}$$

which is in complete agreement with Eq. (12) of Kopeikin (2001). In deriving Eq. (9), we take account of the propagation only of light but not of gravity, since gravity propagation appears at $O(c^{-4})$. Hence, it turns out that the excess time delay given by Eq. (9) is due to nothing but the light-cone effect.

Before closing this section, let us mention recent papers on this issue: Clifford Will reconfirmed my conclusion by explicitly denoting the speed of light and gravity by different characters in his careful computations[11]. Two more papers[12,13] support our conclusion. Nonetheless, Kopeikin wrote two papers [14,15]: In one paper [14], he introduced an artificial time τ associated with the gravity speed c_g, and he claimed again that the excess depended on V/c_g, namely the gravity speed. However, we should notice a key that the velocity V is $dx/d\tau$ but not the coordinate velocity $v = dx/dt$. There exits a relation $ct = c_g\tau$ in his computations, so that in the excess we can replace V/c_g with v/c. The conclusion of the paper thus must be that the excess depends on the speed of light. In the other paper [15], when equations for the gravitational field are integrated, he used the advanced Green function to show that the signature of the excess changed. This signature change apparently means the excess could be related with the gravity propagation. However, even for static case ($v = 0$), signatures of two terms in a logarithmic function change in his expression of the Shapiro time delay, which must be unchanged. It's unbelievable! This happens because he integrated by mistake the null geodesics for light propagation on a future light cone emanating from the observer but not a past light cone. Therefore, if the null geodesic were treated correctly on the past light cone, the signature of the excess could not change for replacing the retarded Green function for gravitational fields with the advanced one. In short, this also is a proof that the excess has no relevance with the gravity propagation. By re-examining all of these recent papers, it seems that our conclusion has been well established.

3 Conclusion

What was measured by the Jupiter event on the 8th of September 2002 is not the gravity speed. Nonetheless, we should look on a positive side: The angular accuracy of their measurement by Very-Long Baseline Interferometry (VLBI) is a few tens micro-arcseconds, about one fifth of previous ones. This has great impacts on astrophysics. One is that a tighter constraint can be put on scalar-tensor theories of gravity through measuring the light deflection with the accuracy. Furthermore, it enables us to determine a parallax distance to stars at a few kpc, and to measure their proper motion by observations during several years. Therefore, the advanced VLBI must play an important role in cosmology as well as stellar and galactic physics.

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References

1. M. Homma, N. Kawaguchi and T. Sasao, Proc. SPIE, **4015**, 624 (2000).
2. S. Kopeikin and G. Schäfer,
3. S. Kopeikin, Astrophys. J. **556**, L1 (2001) Phys. Rev. D **60**, 124002 (1999)
4. E.B. Fomalont and S. Kopeikin, astro-ph/0302294.
5. H. Asada, Astrophys. J. **574**, L69 (2002)
6. C.M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge: Cambridge Univ. Press, 1993)
7. K.S. Thorne, Rev. Mod. Phys. **52**, 299 (1980)
8. P.C. Peters and J. Mathews, Phys. Rev. **131**, 435 (1963)
9. J.H. Taylor, Rev. Mod. Phys. **66**, 711 (1994)
10. I.I. Shapiro, Phys. Rev. Lett. **13**, 789 (1964)
11. C.M. Will, astro-ph/0301145.
12. J.A. Faber, astro-ph/0303346.
13. S. Samuel, astro-ph/0304006.
14. S. Kopeikin, gr-qc/0212121.
15. S. Kopeikin, astro-ph/0302412.