ASSESSMENT OF CANDIDATES' MATHEMATICAL KNOWLEDGE ON ADMISSION TO PRIMARY EDUCATION DEGREES: A REQUIREMENT?

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Our attention is focused on the mathematical knowledge of students enrolling for Primary Teaching Degrees in Catalan universities. We present the preliminary steps of a study[1] which aims at developing a mathematics test that should be included in the official entrance examination for applicants to the teaching degree starting September 2017. After briefly introducing the concept of Basic Mathematical Knowledge (BMK) and determining the content to be assessed in the entrance examination, we present a pilot test conducted on 291 students in order to evaluate their BMK. Our results not only evidence the candidates' BMK inadequacy, but confirm the need to consider mastering BMK as a requisite for admission to the Primary Teaching Degree.

Keywords: entrance examination, initial teacher education, basic mathematical knowledge.

INTRODUCTION

To this date, University entrance exams in Spain are identical for all degrees, without specific tests for each type of study. However, the new legislative change which regulates education in Spain (LOMCE – Ley Orgánica para la Mejora de la Calidad de la Educación, i.e. Organic Law for the Improvement of Quality in Education), passed in November of 2013, established a validation of 'baccalaureate' and allowed university campuses to design their own tests for University entrance. Therefore, it is essential to find a more precise way of establishing what these tests aim to measure for the entrance to each degree offered.

In the case of the degrees in Education, these tests have not yet been defined. However, in Catalonia, both social media and academics are recently paying an increased attention to the need to improve pre-service teacher training. However, those in charge of political decisions need to be convinced that a test on mathematical content knowledge is a necessary part of an entrance examination for accessing a primary teaching degree. Therefore, the first results of our research allows us to stress out that it is necessary to develop a test.

Therefore, in this study, we suggest considering the evaluation of candidates' basic mathematical knowledge (BMK) on entrance to University to start their training in Primary School Teaching. Thus it is not only essential to determine the form and

content of the BMK, but also to prove that candidates' BMK sufficiency cannot be taken for granted.

The TIMMS study (Third International Mathematics and Science Study) evidences differences and deficiencies in the mathematic knowledge of students of several countries. Spain ranked below the average of participating countries of the European Union and the OECD. This fact brings to light the need to revise the teaching of mathematics in the Spanish educational system and suggests that it is paramount to provide a good initial education to future primary school teachers in order to improve this situation.

With results obtained by TIMMS as starting point, TEDS-M (Teacher Education Study in Mathematics) was created, an international comparative study about the knowledge acquired by future mathematics teachers in primary education and compulsory secondary education after their initial training. The aim of TEDS-M was to analyse the differences between initial training programmes and their impact on the education of future teachers. Despite the low number of participating countries in this study and the differences between the training programs of each of them, it brings evidence that better results were obtained in those countries where education in mathematics is more specialised. In this respect, Lacasta and Rodríguez (2013) have documented nominal relations between the level of mathematical knowledge of educators and their level of knowledge for mathematics teaching, mathematical content being the main requisite for a good understanding of how to teach mathematics.

The interest in discovering knowledge for the teaching of mathematics has promoted the evaluation of its content and, particularly, the evaluation of future teachers' knowledge (Norton, 2012; Senk et al., 2012, Walshaw, 2012). However, there is little research describing the mathematical knowledge of students at the start of their training to become teachers, therefore the evaluation of such knowledge is a challenge for their educators (Linsell & Anakin, 2012).

Our objective in this communication is to introduce the concept of BMK and present the results of a research that motivate the need to study it. We present a first theoretical approach to this concept and shortly present how the content required for assessment in entrance examinations was fixed. Our proposal is supported by the expert knowledge of researchers who are also experienced educators of primary school teaching students. Subsequently, we present some partial results from a pilot test designed to evaluate such knowledge that was administered to 291 first year students of the Primary Education Degree at the Universitat Autònoma of Barcelona. Our empirical results allow us to justify the importance of establishing a BMK to be mastered as a requisite to enter teacher training.

BASIC MATHEMATICAL KNOWLEDGE

Shulman (1986; 1987) stressed the importance of content knowledge, defining the latter as the amount and organisation of knowledge of the subject, pointing out that content knowledge requires going further than being familiar with facts and concepts of the subject, it also requires the understanding of its structures. According to Fennema and Franke (1992), Knowledge of Mathematics includes teacher knowledge of the concepts, procedures, and problem-solving processes, the concepts underlying the procedures, the interrelatedness of these concepts, and how these concepts and procedures are used in various types of problem-solving. These authors coincide with Shulman when stating that teachers shouldn't only know mathematical procedures but should also understand the concepts underlying these procedures.

Later on, Ball, Thames & Phelps (2008) used the model suggested by Shulman and elaborated the MKT model (Mathematical Knowledge for Teaching), that was created to describe the knowledge of in-service teachers. One of the main axes of this model is Shulman's Content Knowledge, which they called Subject Matter Knowledge. The MKT model proposes the division of Subject Matter Knowledge in three subdomains: Common Content Knowledge, Specialized Content Knowledge and Horizon Content Knowledge. Common Content Knowledge is the knowledge that every adult that has received mathematical training should have and is used in a wide range of contexts (Ball, Hill & Bass, 2005). Specialized Content Knowledge includes an understanding and mathematical reasoning inherent to the teacher. Horizon Content Knowledge is mathematical knowledge that students will be learning in the future. Again focussing on in-service teachers, Rowland (2008), based on observation in the classroom, proposes the Knowledge Quartet to describe mathematics teachers' knowledge as having four dimensions: foundation, transformation, connection and contingency. In particular, the foundation dimension includes, among others, the propositional knowledge on which teachers support their practice.

In the last decade, several researchers have contributed with nuances or new proposals to the established ideas and have helped to consolidate and expand existing concepts. However, all the concepts introduced in the previous paragraphs refer to teachers' knowledge needed for the actual practice of teaching while we are not even dealing with novice teachers, but with teacher-students. It cannot be expected of students who start their degree to have received a previous education that provided them with a deep understanding of the mathematics concepts studied or an outlook oriented towards conferring their learning to others.

Therefore, in Castro, Mengual, Prat, Albarracín and Gorgorió (2014) we introduced BMK as the disciplinary mathematical knowledge that students need in order to benefit from their courses in mathematics and mathematics teaching during their education to become teachers. It is important to note that we are referring to students that have not even started their training as teachers, and we suggest that BMK should

be a requirement for their pre-service training. BMK would be the initial disciplinary knowledge on which to build throughout teacher students' training, to attain the mathematical and pedagogical knowledge required to start their professional practice. As educators of teachers, we take BMK as the mathematical knowledge starting point for our courses, which should be based on a thorough knowledge of elementary mathematics, being the foundation that would support the building of a structurally robust training.

BMK should be the basis on which to build Shulman's Content Knowledge (1986, 1987) and of Fennema and Franke's Knowledge of Mathematics, but we cannot expect Shulman's idea to be equal to BMK in its entirety. In Ball et al.'s MKT model, BMK is part of common knowledge and the starting point for development of the knowledge of the horizon, since the education the students have received before reaching University level should have allowed them to deal with more advanced knowledge than what they are going to teach in Primary School. Similarly, we believe that we may require our students to know the basis and terminology of the mathematics they have been taught during their previous schooling. Therefore, we consider BMK to be part of the foundation component of Rowland's Knowledge Quartet.

Similarly to the outlook presented in this study, Linsell & Anakin (2013) claim that the models developed to describe the professional knowledge of the teacher have limitations when it comes to the knowledge analysis of beginning undergraduate students. Linsell & Anakin (2012) propose the concept of Foundation Content Knowledge to refer to the knowledge of mathematical content that future educators possess when starting their training programme. This type of knowledge includes as inseparable conditions, both conceptual knowledge and methodological knowledge. The characteristics of Foundation Content Knowledge are related to the ability to model, modify, reason and confirm, the implementation of multiple representations, making generalizations, working with real numbers and understanding basic facts, amongst other aspects.

Our notion of fundamental mathematical knowledge differs little from the Foundation Content Knowledge of Linsell & Anakin (2012), since they refer to the knowledge of future educators at the beginning of their training. However, our research distinguishes itself from that of these authors already at a preliminary stage, given that we wish to determine the knowledge required at the start of undergraduate teacher training, which we have termed BMK. We aim to ascertain the content of the latter by consensus between experts in order to evaluate the BMK. Linsell & Anakin evaluate the knowledge students actually have in order to end up describing it as insufficient, possibly as a comparison to the desired amount of non-explicitly stated knowledge.

METHODOLOGY

After developing a preliminary theoretical approach to Basic Mathematical Knowledge, we set out to establish the mathematical content domains to which it refers. We focus on the mathematical content prescribed by the curriculum of the Spanish compulsory education –Numbers and Arithmetic, Space and Shape, Relations and Change, Measure, and Statistics and Randomness– since we do not expect an encyclopedic knowledge from our students, but wish to verify whether they possess a solid basic knowledge.

In parallel, while developing the criteria to fix the exam content, we set out to elaborate a pilot diagnostic test that should be the first step towards a tool to assess students' BMK. For this purpose we revised different pre-existing tests aimed at the evaluation of mathematical knowledge of teachers in different moments of their training or professional development. Some of the aforementioned tests include TIMMS, TEDS-M, items from the Texas Mathematics Educator tests, as well as the activities employed by Linsell & Anakin (2012) in their study.

These test items are designed with an open-question format to avoid suggesting possible answers to the students, as may be the case when using a multiple-choice question format. The questions aim to evaluate mathematical knowledge at three different levels: reproductive, applicative and relational. Finally, we selected twenty five activities that comprised a balanced test with respect to content blocks and levels of mathematical knowledge. Some of these exercises will be shown later on, together with the results obtained.

The aforementioned test was handed to 291 students of the first year of the Primary Education degree at the Autonomous University of Barcelona (UAB) who had not yet taken any lessons in mathematics or mathematics teaching. The minimum University-entrance examination grade to enter these studies at the UAB is the highest required amongst the 8 degrees in Primary Education of the different Universities in Catalonia. On the other hand, the minimum entrance grade is located at the 81st percentile (77 of 421) in relation to all degrees offered at Catalan Universities. Therefore, we can state that not only have our students successfully passed their educational stages previous to University admission, but have also obtained higher University-entrance qualifications than students entering many other graduate courses.

SOME RESULTS

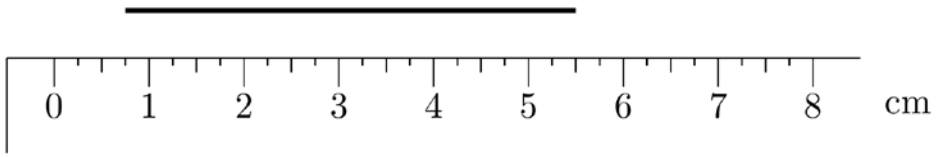
As follows, we present the analysis of some of the data from the answers of the 291 students to three of the questions included in the aforementioned test.

The main interest of our study is focussed on determining the type of background content desired for students recently admitted to the Degree in Primary Education

and diagnosing mistakes made in their learning process. For this reason, we wish to make a quantitative analysis of the type of mathematical content these tests reveal.

Measuring a segment with a ruler

In one of the questions of the test, the students were given the following image and were asked to establish the length of the segment.



The following table summarizes the students' answers to this question:

Answer	Frequency	Percentage	Answer	Frequency	Percentage
NC	5	1.7%	4.8 cm	7	2.4%
4.3	11	3.8%	5 cm	11	3.8%
4.3 cm	11	3.8%	5.3 cm	11	3.8%
4.5 cm	6	2.1%	5.5 cm	4	1.4%
4.6 cm	9	3.1%	5.75 cm	14	4.8%
4.7	4	1.4%	6 cm	4	1.4%
4.75	35	12.0%	Other	53	18.2%
4.75 cm	106	36.4%	Total	291	100.0%

Table 1. Answers to question "Measuring a segment with a ruler"

The data on the table show that the correct answer, 4.75 cm, is also the most frequent one, 35.7% of the students. We also see that 11.2 % of the students give the number resulting from the measurement, 4.75, but they do so without units. For what refers only to the use of units, 75.1% of them use the appropriate ones, 2.8% of the students do not use any, and 0.4% use the wrong units such as cm^3. It is important to note that we have identified 44 different answers for this question, suggesting that the use of open questions does not condition the students' response. However, the most discouraging answers are those where the result given –twice 9.5cm, 19cm, 25.5 and 47– is bigger than the length of the ruler on itself –8cm.

Perimeters and surface areas

One of the test questions asks the students to calculate the surface area and perimeter of a square with 7 cm sides and of a circle with a radius of 6 cm. Table 2 shows the different categories into which we have organized the answers, the number of answers that fall into each of them and the percentage for each category considered.

	Square		Circle	
Answer	Area	Perimeter	Area	Perimeter
No answer	39 (13.4%)	63 (21.6%)	147 (50.5%)	182 (62.5%)
Correct	115 (39.5%)	174 (59.8%)	38 (13.1%)	42 (14.4%)
Correct calculations - wrong units	102 (35.1%)	27 (9.3%)	32 (11.0%)	16 (5.5%)
Interchange area and perimeter	15 (5.2%)	5 (1.7%)	7 (2.4%)	2 (0.7%)
Wrong for other reasons	20 (6.9%)	22 (7.6%)	67 (23.0%)	49 (16.8%)

Table 2. Answers to question “Perimeters and surface areas”

The results in table 2 show high indices of “unanswered” questions, and a clear ignorance of the calculation process for the surface and perimeter of a circle. In particular, it is worth noting that 30 of the students, 10.3% of the total, do not employ the number ‘pi’ for neither the calculation of the circle’s surface area nor for that of its perimeter.

Added to the misunderstandings between the concepts of surface area and perimeter, there are errors in terms of the units used when giving the answers, especially for surface areas. The results obtained when considering only the units in the answers (without considering the calculated figure provided) are shown in table 3.

	Square		Circle	
Answer	Area	Perimeter	Area	Perimeter
N/A	39 (13.4%)	63 (21.6%)	147 (50.5%)	182 (62.5%)
Correct Units	130 (44.7%)	192 (66.0%)	64 (22.0%)	74 (25.4%)
Incorrect Units	70 (24.1%)	1 (0.3%)	25 (8.6%)	2 (0.7%)
Without units	52 (17.9%)	35 (12.0%)	55 (18.9%)	33 (11.3%)

Table 3. Use of units in the answers to question “Perimeter and surface areas”

Contextualised problem with verbal formulation

The formulation of another question of the test is the following: “When going on a school’s outing it is required for children to be accompanied by adults. Each adult can be responsible, at the most, for a group of 16 children. In an outing with 54 children, how many adults are needed to accompany them?”

Table 4 summarizes the answers of the students to this question and shows their relative and absolute frequencies.

Answer	Frequency	Percentage	Answer	Frequency	Percentage
No answer	17	5.8%	4	154	52.9%
3	41	14.1%	4 with errors	20	6.9%
3.375	24	8.2%	5	6	2.1%
3.4	12	4.1%	Other	10	3.4%
3.5	7	2.4%	Total	291	100.0%

Table 4. Answers to question “School’s outing”

We can see that 52.9% of the students answer this question correctly, calculating the ratio in excess, in order to take into account the context of the formulation. However, some of the students give an answer of 4, based on erroneous calculations or invalid arguments (6.9% of the total students). On the other hand, we also find a trend that groups questions 3.375, 3.4 and 3.5, in which the students consider the result of the division to directly be the answer to the question, thus overlooking the interpretation of the situation exposed in the formulation of the problem. These answers represent 14.8% of the total. There is another group of students (14.1%) who give the answer of 3 adults who should accompany the group of children. These students do not consider the possibility of finding the ratio of the division in excess and act without understanding the context of the problem, using a ratio which does not cover the number of children.

CONCLUSIONS

Our students at the start of their Degree in Primary Education at the UAB have successfully passed their studies previous to University but with an incomplete BMK, according to the results obtained in our empirical study. Specifically, we have documented a lack of competence related to BMK, in aspects that have to do with units of measurement, or the contextualisation of mathematical knowledge.

There may be an implicit agreement among teachers of mathematics and mathematics education in Catalonia about what constitutes the BMK required of our students, but this has never been explicitly stated. Therefore, when it comes to the knowledge used as starting point for training in Degrees in Primary Education, it is paramount to clearly set out what is expected of our students. From a teacher-training point of view, results such as those exposed herein evidence the need to commit to the improvement of our students’ understanding of elementary mathematics, in order to successfully face subjects related to its teaching.

The results obtained show that many of our students have not developed an adequate construction of mathematical knowledge during their previous education, and are therefore not able to reproduce those processes they learned by heart and without searching for their meaning within a practical context. Many of the students who enter University may have possibly forgotten the elementary mathematics they once studied. Therefore, we would agree with Fennema and Franke (1992) and Linsell and Anakin (2013) on the fact that the knowledge students carry with them at the start of their training may possibly be characterised by memorising and standardised problem-solving and is far from the one we, as teachers' educators, would expect them to have.

The proof of the lack of elementary mathematical knowledge of students at the start of their teacher training justifies the notion of Basic Mathematical Knowledge and suggests the need to keep working not only towards its characterisation, establishing its form and content in order to evaluate it, but also towards developing and validating a tool to assess the BMK of candidates entering a Primary Teaching Degree. As a conclusion, if we had to give a short answer to the question posed in the title of the paper – Should the assessment of candidates' mathematical knowledge a requirement for on admission to primary education degrees? –our answer would be yes, despite the practical and political implications of taking a decision of such importance.

NOTES

1. Estudi per a l'avaluació diagnòstica de les competències matemàtiques dels estudiants del grau en Educació Primària (Study for the diagnostic evaluation of mathematical competences of students of Primary Education Degrees). (AGAUR Catalonia, ref. 2014 ARMIF-00041)

2. Caracterización del conocimiento disciplinar en matemáticas para el grado de educación primaria: matemáticas para maestros (*C*haracterisation of the disciplinary knowledge in mathematics for the Degree in Primary Education: mathematics for teachers) (DGU, Spain, ref. EDU2013-4683-R).

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