Frameworks supporting the coding and development of mathematics teachers' instructional talk in South Africa

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In this paper, we present two frameworks – for secondary and primary levels - aimed at coding and supporting the development of mathematics teachers' instructional talk in South Africa. Both frameworks draw from sociocultural bases focused on mediating categories within teacher talk, and quality indicators within them, and a press towards mathematics viewed as a network of scientific concepts. Both frameworks provide greater disaggregation, at the lower extreme in particular. This is important in turns of the imperative to support development of teacher talk in a context where disconnection, ambiguity and gaps in teachers' mathematical knowledge are described as relatively common.

Key words: instructional talk; mathematical discourse in instruction, mediating primary mathematics; mathematical quality of instruction; South Africa

INTRODUCTION

Several frameworks are available in the international literature that characterise the quality of instruction in mathematics classrooms. For example, Hill et al's (2008) Mathematical Quality of Instruction framework features aspects like lesson format and links to learning alongside teachers' mathematical talk. For a range of reasons outlined below, our attention, in the context of linked research and development projects aimed at improving the quality of mathematics teaching across ten primary and ten secondary schools in South Africa, is more specifically on the quality of the mathematics that is made available to learn within instruction in mathematics classrooms. The format of instruction and the nature of learner participation therefore fall outside our central scope of attention.

A number of issues - some overlapping and some differing - mark the research and development context when looking across secondary and primary mathematics. A key focus in the overlapping area is on what we call 'mathematical discourse in instruction' (MDI) – which, in parallel with the issues, is articulated in different ways across the two phases. Important differences relate to the much greater use of physical artefacts in primary mathematics, in comparison with secondary mathematics. Working developmentally in secondary and primary mathematics teacher education in this context is premised on our being able to characterise the pedagogic range of MDI at secondary and primary levels on the ground and build from this ground upwards. A focus on teachers' mathematical talk has been central to this focus and a function of a range in the South African context that is broader than is commonly described in the international literature.

Key issues have been identified as concerns relating to teachers' mathematical talk in South Africa. Some of these issues are linked with, and characterised, in frameworks in the international literature base. This is particularly true at the upper end of our concerns across primary and secondary levels where we have episodes of teaching that focus broadly on 'rules without reasons' (e.g. Skemp, 1987), This kind of teaching is widely critiqued in the international literature base as procedural and limiting of access to mathematical discourse. At the lower end though, the international literature contains much more limited disaggregation. Across our work, we have described episodes of teaching in both phases where concerns relate more fundamentally to mathematical coherence. In this teaching, we see episodes that sometimes confirm answers as though they are already known in the classroom space rather than deriving them, teacher talk about knowns as if they are unknowns, and talk that is infused with ambiguity, error and high levels of disconnection (Adler & Ronda, 2017; Venkat & Naidoo, 2012).

While typically, the instructional triad views teaching as mediating between students and the mathematical object in focus, the range of problems identified above, coupled with evidence of significant content and pedagogic content knowledge gaps amongst South African teachers, leads to our attention to the teacher – mathematical object relation as the key initial link to both describe and strengthen in order to support teaching development on the ground.

The range overviewed above meant that we needed frameworks that allowed for adequate description and categorization of the ground. This involved the identification of key categories within instructional talk, and characterising quality markers that could also serve as developmental pathways within these categories. In this paper, we present and discuss the categories of instructional talk that we have focused on within MDI[1] at secondary and primary levels, and the quality markers within them. Looking across the two framings of mediating talk, we comment on the ways in which they are linked by a concern with incoherence and error at the lower extreme, and with mathematics viewed as a network of scientific concepts at the upper extreme, with focus on structural relations and generality as key indicators of mathematics worked with in these ways within instruction. We go onto present episodes of teaching drawn from the lower and upper level of concerns and outline our ways of coding them using our respective coding frameworks.

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[1] In subsequent writing, we have shifted to referring to the framework for MDI at primary level as the 'Mediating Primary Mathematics' or MPM framework in order to avoid confusions between the primary and secondary models. We have retained the MDI-S and MDI-P terminology for this paper to retain historical accuracy at its point of writing.

MDI FRAMEWORKS

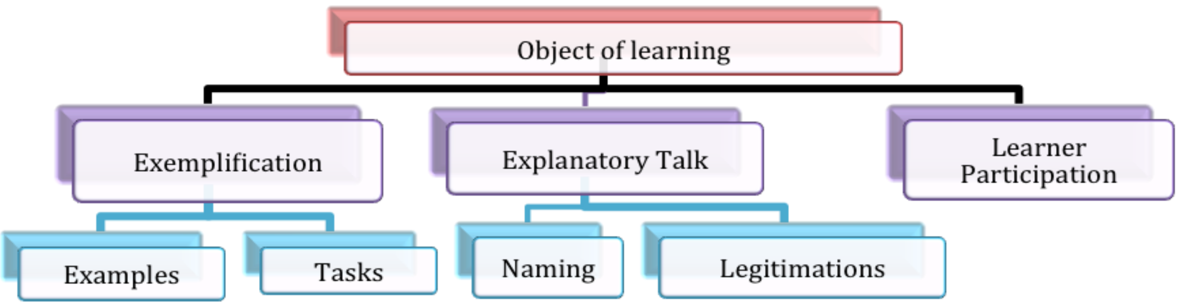
Across both phases, the concerns outlined earlier led to an emphasis on the view that that learning is always about something. Bringing into focus what this is, in terms of what learners are expected to know and be able to do, is central to the work of teaching. Marton and Tsui (2004) refer to this 'something' as the object of learning: 'The object of learning … is defined in terms of the content itself … and in terms of the learner's way of handling the content' (p. 228). Foregrounding the connection between 'object' and 'learning' is central, and contrasts with lesson 'goal' formulations. An object of learning in a mathematics lesson could be a concept, procedure or algorithm, or meta-mathematical practice. It goes without saying that the object of learning needs to be in focus for the teacher.

Juxtaposing primary and secondary level frameworks allows us to highlight ways in which the two frameworks differ in the aspects they focus on within their overall commonalities of focus on the mediation of mathematics predicated on the need for structure and generality.

MDI-Secondary (MDI-S) and mediating talk

In the MDI-S framework captured in Figure 1, the key generative mechanisms for the work of teaching are exemplification, explanatory talk and learner participation (for detail see Adler & Ronda, 2015). What stands between (i.e. mediates) the object (and here of learning) and the subject (the learner) are a range of cultural tools: examples and tasks, word use and the social interactions within which these are embedded. In this paper, our focus is on teachers' explanatory talk and how we think about quality within its two key features: naming and legitimating criteria.

Figure 1: Constitutive elements of MDI-S and their interrelations



Explanatory talk

Our emphasis on explanatory talk draws on Bernstein's (2000, p. 36) notion of evaluation[2]. For Bernstein, any pedagogic discourse, and hence the discourse in mathematics lessons, transmits criteria as to what counts as mathematics. The transmission of criteria occurs continuously, be it implicitly or explicitly, through

[2] Bernstein's notion of 'evaluation' is not to be conflated with assessment.

messages that are communicated as to what is valued with respect to the object of learning i.e. what is to be known or done, and how. We call this *explanatory talk*[3], the function of which is to name and legitimate what is focused on and talked about i.e. related examples and tasks. Analyzing how objects[4] focused on are named, and what is legitimated in an episode is key to being able to describe the mathematics made available to learn through explanatory talk, as well as reach a summative judgment on naming and legitimating as these accumulate over time in a lesson.

- ***Naming***

Learners' encounters with mathematical objects also occur through how these are named. We define naming to mean the use of words to refer to other words, symbols, images, procedures, or relationships, in the course of instruction. The tension in managing both formal and informal ways of talking mathematically, and thus naming what is focused on in class is now widely recognized. In WMC-S, we noticed some teachers' reluctance to use formal mathematical language as it is "abstract and the learners are put off", and others' over reliance on formal talk with neglect of connecting mathematical ideas to colloquial meanings.

We categorise naming within episodes as either *colloquial / non-mathematical* (and here we include *everyday language* e.g. 'over' in division, and/or *ambiguous pronouns* such as this, that, thing, to refer typically to what is being pointed to on the chalkboard) or *mathematical*. In this latter category we distinguish *mathematical words* used as *labels* or *name on*ly e.g. to read a string of symbols from *formal mathematical language* used. For example, in the first lesson extract below, *transpose* is categorised as *non-mathematical*, despite its common use in our mathematics classrooms. This is not because the word *transpose* should not be used when solving equations and inequalities. Our point is simply that if this is used exclusively to describe an algebraic transformation, with no accompanying mathematical justification (e.g. we subtract 6 from both sides of the equation) then underlying principles or properties like maintaining equivalence are never made explicit. Our purpose is to see the extent of *both* colloquial and formal mathematical talk and *the movement between these*.

- ***Legitimating criteria***

We distinguish criteria of what counts (or not) as mathematical that are particular or localized, or call on memory (L) (e.g. a specific or single case, an established shortcut, or a convention) from those that have some generality (e.g. equivalent representation, definition, previously established generalization; principles, structures, properties),

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[3] The name here draws attention to the mathematical quality of the explication or elaboration offered – we could equally have named this explicatory or mediating talk.

[4] Our use of 'object' here is in the most general sense and includes all that is in focus e.g. words, symbols, images, pictures, material objects, etc.

distinguishing partial (PG) e.g. variables described as “letters which represent numbers which we do not know”; from full generality (FG) e.g. variables described as “letters representing any number”. We are also interested in non-mathematical criteria (NM), everyday knowledge or experience (E), visual cues (V) as to how a step, answer or process ‘looks’ (e.g. a ‘smile’ as indicating a parabola graph with a minimum, or memory devices that aid recall (e.g. FOIL)); or when what counts is simply stated, thus assigning authority to the position (P) of the speaker, typically the teacher. We further indicate errors in legitimating talk, which fell largely within NM by a negative sign e.g. V- .

The significance of these varying criteria is the opportunities they open and close for learning. Most obvious are the extremes of legitimations based on the one hand on principles of mathematics, thus with varying degrees of generality, and possibilities for learners to reproduce or reformulate what they have learned in similar and different settings. On the other hand, appeals to the authority of the teacher) and/or visual cues produce a dependency on the teacher, on memory (this is what you must do); or on how things ‘look’, requiring imitation that is local or situational (Sfard, 2008). While imitation might be necessary in aspects of mathematics learning, these cannot be the endpoint of learning. The criteria for what counts as mathematics that emerge over time in a lesson are thus key to what is made available to learn in terms of movement towards scientific concepts.

Table 1 summarises the categories and coding for explanatory talk. The categories themselves do not form a hierarchy – they distinguish different kinds of talk that emerge over a lesson in varying ways. In the second row are the levels we assign when we look at the accumulating categories across a lesson. The levels are hierarchical and reflect our privileging of mathematical names and principled criteria. We emphasise here that the assignment of a level in our analysis is an interpretive judgment, reflecting our privileging of generality through exemplification, mathematical names and principled criteria, and as these unfold over a lesson.

Figure 2: Explanatory talk – MDI-S

Explanatory talk	
Naming	**Legitimating criteria**
Within and across episodes word use is: *Colloquial* **(NM***)* *e.g.* everyday language and/or ambiguous pronouns such as this, that, thing, to refer to objects in focus *Math words used as name only* **(Ms)** e.g. to read string of symbols *Mathematical language used appropriately* ***(Ma)*** to refer to other words, symbols, images, procedures	Legitimating criteria: *Non mathematical* ***(NM)*** *Visual* **(V)** – e.g. cues are iconic or mnemonic*;* *Positional* ***(P)*** – e.g. *a* statement or assertion, typically by the teacher, as if ‘fact’. *Everyday* ***(E)*** *Mathematical criteria:* *Local* **(L)** e.g. a specific or single case (real-life or math), established shortcut, or convention *General* **(G***)* equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial **(GP)** or ‘full’ **(GF)**

Use of colloquial and mathematical words Level 1: NM – there is no focused math talk – all colloquial/ everyday Level 2: movement predominantly between NM and Ms, some Ma Level 3: movement between colloquial NM, Ms & formal math talk Ma	Criteria for what counts as mathematics that emerge over time in a lesson and provide opportunity for learning geared towards scientific concepts. Level 0: all Criteria are NM i.e. V, P, E Level 1: criteria include L – e.g. single case, short cut. Level 2: criteria extend beyond NM and L to include Generality, but this is partial GP Level 3: GF math legitimation of a concept or procedure is principled and/or derived/proved

Episodes and their analysis[5]

Secondary – Episode 1, lower end

Solving quadratic inequalities is included in the Grade 11 curriculum. For the inequality $x^2 > 4$, the teacher instructed the class that to solve for x, we “do exactly the same” as the steps followed in solving the equation $x^2 = 4$. After “transposing 4” to obtain $x^2 - 4 > 0$, “you then factorise to obtain $(x - 2)(x + 2) > 0$.” He then wrote $x - 2$, looked to learners some of whom called out “greater than”, and completed the inequality $x - 2 > 0$. He then asked the class whether he should write ‘and’ or ‘or’ and while some learners called ‘and’ and others ‘or’, he said “I will take ‘and’”. He continued with $x + 2$ and with some learners offering ‘less than’, and others ‘greater than’, he wrote x < -2 and said “x is less than negative two”. The answer produced on the board was $x > 2 \; and \; x < -2$. Having produced this answer, he then asked learners: “Now how come is it that the sign changed?” (pointing to > in the second part of the answer) and worked with learners testing various numbers to confirm the two inequalities.

Naming: With the exception of “this ‘and’ or ‘or’ thing” (and so demonstrative pronouns) both the teacher and learners used mathematical words as *labels or to name the symbol strings* they were talking about, hence coded as follows: **(Ms).**
While in MDI-S we do not level an episode, for our purposes here, mathematical words are used, but only for labeling or reading symbol strings. If this persisted through the lesson, naming would then be **level 2.**
Legitimating criteria: The legitimating talk accompanying the steps taken to write down the answer $x > 2 \; and \; x < -2$ for the inequality $x^2 > 4$ were at the level of assertions with no rationale for obtaining the inequality relations, nor the erroneous connector ‘and’. (**P**$^-$) The interpretive judgment, if restricted to this episode, would be that the legitimation was by assertion, and erroneous, and so NM and **level 0**. While the teacher proceeded to test various numbers, these were used to confirm an asserted solution and not to derive it.

Secondary – Episode 2, more familiar

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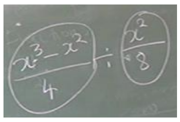
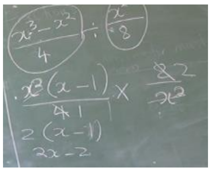
[5] Both episodes have been described previously in papers differently focused, and where they form part of a full lesson analysis.

In a Grade 9 lesson introducing the division of algebraic fractions, the teacher used $\frac{2}{6} \div \frac{2}{3}$ as a first example to recall the rule "change the sign and swap over". The same rule was applied to $\frac{2x}{6x} \div \frac{2x}{3x}$ and then she put up the third example $\frac{x^3 - x^2}{4} \div \frac{x^2}{8}$ and said : "It's one and the same thing. They give you something like this (writes symbols on board), ok? … Over here (points to $\frac{2}{6} \div \frac{2}{3}$) you just have two numbers, a fraction divided by a fraction, ok? (Learners chorus 'yes'). Over here (pointing back to example 3) is the same thing. I've got, here's one fraction divided by one fraction (circles each fraction).



She then asked learners what they needed to do to complete the division, and continued "… before you divide you factorise, because over here it concerns the common factor. Why? Because we want to have one, one term at the top and one term below, ok?"
After completing the steps illustrated on the right, she concluded: "you just apply the same principle, it's just that when it looks complicated just pause and say what must I do here?" Together with contributions from the learners, she says we "take out the common factor x squared and we get x squared bracket x minus 1 close bracket" and she writes: $x^2(x-1)$. The class continues to call out with her the next steps i.e. "change the sign and swap", and then "cancel common factors"



Naming: In this episode, non-mathematical talk **NM** through use of ambiguous pronouns (e.g. this), was accompanied by mathematical words used mainly to read strings of symbols (x squared bracket x minus 1 close bracket) **Ms**. There was also some appropriate formal naming of objects (e.g. a fraction divided by a fraction, one term, common factor) **Ma.** This episode, again with the limitation that we do not assign levels to episodes, would be **Level 2**.
Legitimating criteria: The overarching **legitimating criteria** in this episode were to previous examples as the 'same thing' and their general structure – one algebraic fraction divided by another **(GF).** The "top" and "below" **(V)** of the fractions were pointed to as each needing to be "one term", and so expressed as factors which were defined in Episode 2 as "dividing without remainder" **(GF).** The division follows a short cut **(L)** (remembered from previous work … change signs and 'swap') with rules and procedures (factorise first, take out common factor, I cannot just go and say …) that were stated, not derived **(P).**
In overview, the criteria for recognizing the *form of the expression* were general, but the criteria for the procedure for division were dominantly localised, as there was reliance on rules, shortcuts, and in some cases assertions by the teacher. Hence, again with the limitation that it is a single episode, as there is some generality at least at the level of form, we would assign this as **Level 2**.

MDI-Primary (MDI-P) and Mediating Talk

Mediating for mathematical learning in relation to focal objects, and with a drive towards mathematics viewed in terms of a connected network of scientific concepts, was central to our work as well, but the key analytical foci, for better fit with the early primary years where much of our dataset was located, differed. In the primary years, a broad swathe of evidence points to the importance of using situations, diagrams, and physical artefacts to provide strong visualizable and imaginable underpinnings for the more abstract symbolic mathematical language that is to come. Mediating for ***connection*** is central to this work, with *physical artefacts*, *inscriptions*, and *talk* then being the key empirical phenomena in the context of tasks and example spaces for examining the nature and extent of connections seen in teachers' MDI. We look, across these phenomena, for features related to the extent to which mathematical structure and generality are made available for appropriation in instruction.

As with MDI-S, we focus specifically on the ways in which mediating talk is categorised, and the markers of quality developed within each of the MDI-P talk categories. The categories we have focused on relate to: generating solutions; building mathematical connections; building learning connections through explanation and evaluation. These categories and the quality markers within them are detailed in Figure 3.

The 'generating solutions' category is focused on teachers' problem-solving methods and strategies within the task and example space in that episode. The hierarchy in this category marks, at the lower end, some of the problems outlined earlier with incoherence and disruptions to mathematical problem-solving processes. At the upper end, quality is viewed in relation to the offer of methods of solving that have generality beyond the example space being worked with, and without restriction to the particular artefact or inscription being worked with in that episode.

Figure 3: Explanatory talk and gesture – MDI-P

Method for generating/ validating solutions	Building mathematical connections	Building learning connections: explanations and evaluations - of errors/ for efficiency/ with rationales for choices
No method or problematic generation/validation 0 Mixing of knowns and unknowns	Disconnected and/or incoherent 0 Disconnected /incoherent treatment of examples OR Oral recitation with no additional teacher talk	Pull-back 0 Pull back to naïve methods OR No evaluation of incorrect offers
Singular method/validation 1 Provides a method that generates the immediate answer; enables lr to produce the answer in the immediate example space	Every example treated from scratch 1	Accepts/evaluates offers 1 Accepts lr strategies or offers a strategy OR Notes or questions incorrect offer
Localized method/validation 2 Provides a method that can generate answers beyond the particular example space	Connect between examples or artefacts/ inscriptions or episodes 2	Advances or verifies offers 2 Builds on, acknowledges or offers a more sophisticated strategy OR Addresses errors/ misconceptions through some elaboration, e.g. 'Can it be ----?' 'Would – this be correct, or this?' Non-example offers
Generalized method/validation 3	Vertical and horizontal (or multiple) connections made between examples/	Advances and explains offers 3 Explains strategic choices for efficiency moves

Provides a strategy/method that can be generalized to both other example spaces AND without restriction to a particular artefact/inscription	artefacts/ inscriptions / episodes 3	OR Provides rationales in response to learner offers related to common misconceptions OR Provides rationale in anticipation of a common misconception

The 'building mathematical connections' category is focused on the ways in which examples, in that episode's example space, are connected within instruction. At the lower end, disconnected/incoherent treatment of examples within episodes, or episodes involving oral recitation pupil responses (relatively common in a context where chorused chanting of answers is relatively common) with no teacher talk, are represented. At the upper end, multi-directional connections within the example space – which is treated as a linked set in the ways described in Watson & Mason's (2006) work, and focused on structure and generality, are aimed at.

In the 'building learning connections: explanations and evaluations' strand, our attention is on instruction focused on progression and explanation – teaching that presents mathematical discourse as having both progressions and rationales. Much of the coding in this strand is seen in the empirical space of teacher responses to learner offers. At the lower end, teaching that 'pulls back' towards more naïve strategies, or fails to offer any evaluation of learner inputs, is described – with both of these phenomena described in South African writing (see Ensor et al, 2009, for the former, and Hoadley, 2006, for the latter). At the upper end, instructional talk works to advance mathematical offers, and provide rationales for choices of steps.

Episodes and their analysis

Primary – Episode 1, lower end

Halving' is the topic being dealt with in a Grade 2 class. Initially, learners are given boxes/bottle tops and asked to make half of 12, 10, 8 and 4. In the following exercise, with bottle tops still available, learners are asked to work out half of the following numbers: 2, 4, 8, 16, 22, 24, 26, 32. In fieldnotes, the observers note that in the early examples, some children appear to 'know' the answer, but have trouble with halving two-digit numbers. The teacher steps in to explain how to work out 'Half of 26'. Each student pair in the class is asked to make 26 balls from clay – which they do taking extended time and, predictably, making balls of different sizes. The teacher draws 26 circles on the board in a line. Her explanation for how to work out half of 26 proceeds as follows: 'I want us to count to 13, and move those balls aside (marks a divide on the board). How many balls are on the other side? 13 as well. So 13 is half of 26.'
Method for generating/validating solutions: 0 (teacher's explanation introduces the solution, 13, at the outset of the problem-solving process, and then verifies its correctness, rather than working with given quantities to deduce the unknown) **Mathematical connections: 0** (through much of the episode, there is no additional teacher talk relating to the example space; where talk comes in, the example is dealt with in incoherent ways described above) **Learning connections: 0** (no evaluation of learner working in this episode)

Primary – Episode 2, more familiar

Within a lesson focused on working on place value based 'breaking down' and 'building up' of numbers,

the first episode with this focus (following some work on counting and number bonds) involves a task asking the class to 'break down the numbers: 13, 19, 27, 45, 67, 93, into their place value by quantity, and following this being written up for all examples, then represent the tens and units quantities with ten strips and unit squares on the board'. Learners' offers of the symbolic breaking down are written in by the teacher on the board: (e.g. 13 = 10 + 3). The teacher's associated commentary included emphasizing the horizontal equivalences in each example, and working with the example space as a set to note that: 'we have two digits this side (gesturing down the 'tens' break down values), and 'now the remainder is one' (gesturing down the 'units' break down values). Multiple learner offers across this episode all involve correct answers, but teacher incorporates checks of these offers in two instances through making a counter-offer and asking learners to explain their choices e.g. when a learner states that 'one ten' strip is needed for 19, the teacher picks up one unit square, asks if this is okay, and then probes why not.
Method for generating/validating solutions: 2 (while the methods offered for generating solutions are coherent and fit the example space, this talk would not generalise beyond two-digit numbers, and would also not deal well with either single-digit examples or multiples of ten where the breakdown need not necessarily have a 'ones' component) **Mathematical connections: 3** (horizontal and vertical connections made consistently) **Learning connections: 3** (teaching proceeds smoothly in alignment and with elaborations of learner offers; a common misconception is anticipated in her offer of a unit square instead of the ten-strip suggested by the learner, with probing of why the teacher's choice is incorrect)

DISCUSSION

Our focus in developing and using our frameworks is on the quality of mathematics made available in the classroom MDI. This focus contrasts with the broader scope in frameworks such as Hill et al's (2008) Mathematical Quality of Instruction where features like lesson format and links to learning are incorporated alongside teachers' mathematical talk. Our narrower focus includes more disaggregation between the two levels of concern (incoherence and error at the lower extreme, and structural relations and generality at the upper extreme). Thus, while across both frameworks, categories are theoretically informed, the levels within them are empirically derived with a view to allowing description across the pedagogic range. We have needed key indicators of mathematics worked with across this wider range in instruction than is typical in available frameworks in the international literature. This disaggregation assists with our goals for being responsible in our coding of what is present in instruction, and then being able to be developmentally responsive in our work with teachers.

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