The didactician as a model within classroom activities: investigating her roles

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In this paper we address the issue of teacher professional development, with reference to how to support teachers in activating effective and aware approaches to be adopted during the lessons to foster the students' use of algebra as a thinking tool. We hypothesize that the didactician, intervening during class activities, could act as a role model for the teacher. We analyse a teaching episode, by means of a combined theoretical framework, to highlight, on one side, the way the didactician acts as a model and, on the other side, moments of achieved harmony between the didactician's and the teacher's interventions.

Key-words: teacher professional development, didactician's role, co-learning, classroom interaction.

INTRODUCTION

Since the nineties, research studies have pointed out that algebraic language should be presented and treated in classroom as a tool for representing, exploring relationships, interpreting and developing reasoning (see, as paradigmatic example, Arcavi, 1994). In tune with these research studies, both the authors have investigated the design and implementation of activities of proof construction through algebraic language (Cusi & Malara, 2009; Morselli & Boero, 2011) aimed at promoting algebra as a tool for thinking (Arzarello, Bazzini & Chiappini, 2001).

Few studies have focused on the role played by teacher's actions and interventions in fostering an effective and aware development of reasoning by algebraic language and on the interrelations between these roles and the thinking processes developed by the students. In (Cusi & Morselli, 2016) we addressed this issue, combining two theoretical lenses - the construct of "Model of aware and effective attitudes and behaviours" ($M_{AE}AB$) and Habermas' construct of rational behaviour - in the analysis of a class discussion from a teaching experiment. The analysis showed that the teacher is crucial in catching and deepening occasions of meta reflection, so that students may become aware of their rational behaviour and share it with their mates.

Aware of the complexity connected to the teacher's task of acting as a model in the effective use of algebra as a thinking tool and in promoting students' rational behaviour, we turned our reflection to possible ways to promote the teacher's development on this issue and we focused on situations where teacher and didactician (we use this term in the sense of Jaworski, 2012) collaborate in all phases of the teaching and learning process, from the planning to the implementation and analysis of teaching sequences. Our methodology of working with teachers involves an active role of the didacticians in fostering teachers' analysis of their practice and the use of specific theoretical constructs as tools to support this joint analysis and the communication between teachers and didacticians (Cusi & Malara 2016). In this paper we focus on another important moment in which the teachers and the didacticians interact, that is when the didactician participates to classroom activities. In particular, we are interested in studying the ways in which the didactician could behave, during classroom activities, to foster the teacher's aware activation of the roles that could be played to support students in the use of algebra as a thinking tool.

In the following, we organize our theoretical framework in two sections: at first we illustrate relevant references on the relationship between theory and practice and the possible collaboration between teachers and didacticians to frame our methodology of work with teachers; afterwards we present the theoretical tools we combine to study the actions and interventions of the didactician and the influence of the didactician's actions and interventions in terms of teacher's activation of the different roles that could be played to support students in the use of algebra as a thinking tool.

TEACHERS AND DIDACTICIANS WORKING IN COLLABORATION: THE INTERPLAY BETWEEN THEORY AND PRACTICE

In the last years there has been an increasing interest towards the crucial role played by collaborative ways of working with teachers within teacher education processes. The model of collaboration to which we refer is the one introduced by Jaworski (2003), who has stressed the value of, on one side, fostering teachers' critical reflection about their practice, and, on the other side, sharing these reflections between didacticians and teachers within a community of inquiry. She stresses that this kind of research programs foster the *co-learning* for all the participants: "in co-learning, the learning of one is dependent on the participation and learning of others: mathematics teachers and educators learn together with different roles, goals and learning outcomes, while engaged in common activity for mutual benefit" (Wagner, 1997, quoted in Jaworski, 2003, p. 250). We put ourselves in a perspective of co-learning, since, in this work, we, as didacticians, are reflecting on our roles of teacher educators within the teacher education program in which we are involved.

Jaworski (2012) suggests that, in order to reflect elements of learning and development for teachers and didacticians, the usual didactic triangle (teacher-student-mathematics) should be extended to a didactic tetrahedron (the didacticians representing the fourth vertex), the *expanded didactic triangle*. The expanded didactic triangle enables to focus both on: (a) the traditional didactic triangle, which characterises elements of the relationships involved within a community of teachers, their students and mathematics; (b) a meta-level triangle, which highlights the developmental processes that involve teachers and didacticians. In this paper we will adopt the model of the didactic tetrahedron to describe the focus of our research.

As stated above, we are interested in studying how the actions and interventions of the didactician during classroom activities may influence the teacher's activation of different roles to support the use of algebra as a thinking tool in their students. In particular, we claim that the didactician's interventions during teaching experiments could represent a fundamental way of supporting teachers in activating effective and aware approaches to be adopted during the lessons. This perspective is in tune with Mason's (2008) stress on the teacher educators' role in directing teachers' attention toward constructs, theories, and practices that can inform and guide their future choices, in order to lead them to become aware "not simply of the fact of different ways of intervening, but of the fact of subtle sensitivities that guide or determine choices between types and timings of interventions" (2008, p. 49). In tune with Mason's description of what happens to a student who internalizes the stimuli received by his/her teacher, we claim that, in the same way, the interventions of the didactician during the teaching experiments could foster shifts of attention for teachers and their internalization of the received stimuli, so that the activity of reflection moves from a process "in themselves" to a process "for themselves".

THEORETICAL TOOLS FOR THE ANALYSIS OF THE ROLE PLAYED BY THE TEACHER WITHIN CLASSROOM ACTIVITIES

The $M_{AE}AB$ construct is the result of a study aimed at highlighting the delicate role played by the teacher in effectively guiding his/her students to the construction of reasoning through algebraic language. A set of roles (summarised in the following table) have been identified (Cusi & Malara, 2009, 2016) to outline the approach of a teacher who consciously behave constantly aiming at "making thinking visible" (Collins et al., 1989), in order to make his/her students focus not only on syntactical or interpretative aspects, but also on the effective strategies adopted during the activity and on the meta-reflections on the actions that are performed.

A first group of roles are those performed when the teacher tries to carry out the class activities posing him/herself not as a “mere expert” who proposes effective approaches, but as a learner who faces problems with the main aim of making the hidden thinking visible, highlighting the objectives, the meaning of the strategies and the interpretation of results.	*Investigating subject and constituent part of the class in the research work being activated*: when the teacher asks students to give suggestions about how to go on with the activity, intervening with the aim of making them feel involved in the activity as a group;
	Practical/ Strategic guide: when the teacher poses herself, in front of the problem, as an inquirer who aims at sharing the thinking processes and discussing the possible strategies to be activated;
	“Activator” of interpretative processes: when the teacher makes the students activated proper conceptual frames (Arzarello, Bazzini & Chiappini, 2001) to interpret the different algebraic expressions constructed when solving a problem;
	“Activator” of anticipating thoughts (Boero, 2001): when the teacher makes the objectives of the manipulation of algebraic expressions explicit and recall them during the discussion, in order to enable the students to share these objectives, monitor and control the strategies;

The second group of roles refers to the phases during which the teacher becomes also a point of reference for students, to help them clarify salient aspects at different levels, with an explicit connection to the knowledge they have already developed.	*Guide in fostering a harmonized balance between the syntactical and the semantic level*: when the teacher makes the students focus on the importance of controlling both syntactical and interpretative aspects and she discusses possible problems arisen when the syntactical or the interpretative level is not controlled;
	Reflective guide: when, in front of a student who proposes an effective approach to the resolution of a problem, the teacher asks him/her to make his/her thinking processes explicit, or she repeats what has been said by the student stressing on the reasons subtended to his/her approach, or she asks to other students to interpret what he/she said;
	“Activator” of reflective attitudes: when the teacher poses meta-level questions aimed at making the students evaluate the effectiveness of a strategy and reflect on the effects of a choice that was made during the resolution process.

Table 1: Characterisation of the roles played by a teacher as a $\mathrm{M_{AE}AB}$

The second theoretical tool to which we will refer in our analysis is Habermas' construct of rationality. Drawing from this construct, Morselli & Boero (2011) propose that the discursive practice of proving encompasses: an epistemic aspect (conscious validation of statements according to shared premises and legitimate ways of reasoning); a teleological aspect (conscious choices to be made in order to obtain the aimed product); a communicative aspect (conscious adhering to rules that ensure both the possibility of communicating steps of reasoning, and the conformity of the products (proofs) to standards in a given mathematical culture). When proving through algebraic language, epistemic rationality consists of modeling requirements, inherent in the correctness of algebraic formalizations and interpretation of algebraic expressions, and systemic requirements, inherent in the correct application of syntactic rules of transformation; teleological rationality consists of the conscious choice and management of algebraic formalizations, transformations and interpretations that are useful to the aims of the activity; communicative rationality consists of the adherence to the community norms concerning standard notations, but also criteria for easy reading and manipulation of algebraic expressions. The student must combine the adherence to syntactical rules on one side, and the goal-oriented management of the processes of formalization, transformation and interpretation, on the other. Still related to teleological rationality, the student must be aware of the fact that proving by algebraic language means deriving from algebraic manipulation a new algebraic expression, whose interpretation gives new information concerning the truth of the statement.

RESEARCH QUESTIONS AND RESEARCH METHODOLOGY

The aforementioned theoretical tools were already used to analyse the teacher's complex role as a model for fostering students' rational behaviour when dealing with algebra as a thinking tool (Cusi & Morselli, 2016). We plan to analyse the didactician-teacher interaction and the teacher professional development throughout a 10 years process. The objective of this long-term study will be to analyse the teacher's development, highlighting the ways in which the didactician, collaborating with the teacher, may promote the teacher's awareness of her role in the classroom and, more in general, her professional development.

In this paper we start this analysis, focusing on the didactician-teacher-students (D-T-S) interaction during classroom activites. The model of the didactic tetrahedron (Jaworski, 2012) is helpful in describing the focus of our research. In particular, it enables to describe the complexity of the interactions that our methodology of work with teachers involves. In addition to the traditional didactic triangle (T-S-M) and the meta-level triangle (D-T-M), in fact, the other facets of the tetrahedron introduce new levels at which our analysis can be performed: the triangles D-S-M and D-T-S, in fact, highlight the levels of the interaction between the didactician, the teacher and the students during classroom activities. In this work, our aim is to investigate the ways in which the dynamics that can be analysed looking at the triangles D-S-M and D-T-S may influence, on one side, the interaction between the teacher and her students (triangle T-S-M) and, on the other side, the developmental processes highlighted through the meta-level triangle. For this reason, we will use our theoretical tools for a double aim: studying the way the didactician acts as model for the students, and studying the way teacher is influenced buy the model of the didactician. We use the construct of rational behaviour to discuss the rational behaviour in using algebra as a thinking tool during the activity, and the $M_{AE}AB$ to analyse the role of the didactician as a model for the students. More specifically we focus on moments during which the didactician, thanks to the activation of specific roles connected to the $M_{AE}AB$ construct, fosters the shift of attention in the teacher, who, consequently, tries to activate the same roles. It is a preliminary analysis, mainly aimed at investigating the use how our theoretical tools to highlight these dynamics. We, in particular, focus on an episode during which the didactician and the teacher orchestrate collaboratively a mathematical discussion. Data at disposal are video recordings of the classroom discussion, pictures of the whiteboard, students' written productions.

ANALYSIS OF AN EPISODE

The context we refer is that of the long-term project "Language and argumentation" (Morselli, 2013), aimed at designing and experimenting task sequences with a special focus on argumentation and proof. Within the project, the didactician and the team of teachers collaborate in task design and process analysis. The didactician takes part to all the class sessions, co-conducting the lesson with the teacher. After each lesson there is a brief meeting between the teacher and the didactician, so as to comment the session and plan possible variations for the subsequent session. Regular meetings with all the team of teachers are organized, so as to analyse the processes and compare the teaching experiments in the different classes.

The episode comes from a teaching experiment performed in grade 7. The teacher, who holds a university degree in Chemistry, had more than 10 years of experience in teaching mathematics at lower secondary school level. She was at her fifth year of collaboration with the didactician within the project. She had already taken part to the design and implementation of task sequences for the first approach to algebraic language as a proving tool, but she was at her first experience with the task at issue. The students already had performed some activities on argumentation and first approach to algebra as a proving tool.

Students worked in group on the following task: "*What can you tell about the sum of three consecutive numbers?*". In the subsequent class discussion, the groups shared their answers and explanations with all the class. Only one group (Edel, Sonia and Giulia) attempted an argumentation with letters, proposing two different algebraic representations (the second being an amendment of the first one): *n+n+n=n/3; n1+n2+n3=n/3.* Next to the two expressions, the group proposed a verbal explanation: "*Three consecutive numbers can be added and the result is multiple of 3. The sum of these numbers is divisible by 3 because the added numbers are 3. The middle number is given by the division of the sum of the three numbers*". The following excerpt refers to the discussion on their solution, with a specific focus on the algebraic representations. This solution was presented after another group expressed its conjecture (the sum is always divisible by 3) and proposed a pragmatic explanation, made up of numerical examples.

The discussion starts with Edel, one of the elements of the group that proposed the algebraic expressions, writing at the blackboard the expression n+n+n=n/3 .

1 Edel: I do number plus number plus number, equal n divided by 3.

Bos raises his hand and starts criticizing, but the teacher stops him.

4 D (didactician): I ask you a question: this thing that you wrote … did you write it to express the property or to justify, to motivate it?

5 Edel: To try and explain what we did, to try and explain the way three consecutive numbers can be summed up and give a number that is divisible by 3. To try to explain what we did before, that is the three numbers, the numbers are three and then this is why they are divisible by 3.

6 D: Ok. After, we will reason on her representation. In the meanwhile, what can we find of really different from what we wrote before? (*R is referring to the pragmatic explanation proposed by the previous group*) …That she does not use…

D, referring to the activities performed during the previous school year, guides the students highlighting that the use of letters enables to reason in general terms.

13 Edel: At first we had written number plus number plus number, without 1,2,3, but after one had to add 1,2,3 in order to show that they are consecutive.

14 D: Ok, in order to show that they are different and you say, if we call them n1, n2, n3 I give the idea that they are three consecutive numbers. Ok.

Another student, Alb, proposes to use the two expressions $n\cdot 2+ n\cdot 2+1+ n\cdot 2$ and $n\cdot 2-1+ n\cdot 2+ n\cdot 2+1$*, which are written on the whiteboard.*

18 T (teacher): Ok, this is when we start with an even number, the other one when we start with an odd number.

19 D: What do you think about this proposal, in comparison with the former one?

D guides Alb in making the meaning of the two expressions explicit. Alb, helped also by T, stresses that the two expressions represent two different cases: when the first number is even and when it is odd.

24 D: What do you think of this representation? Do you find it convincing?

25 Vic: It doesn't specify that they are consecutive.

26 D: It does not specify that they are consecutive, that is to say if I get into the room right now and I see the sum written on the whiteboard, do I understand that it is the sum of three consecutive numbers?

27 Vic: It is the sum of an even number plus an odd number or an odd number plus an even number.

28 Alb: You can write first, second and third. As we said before.

29 T: In this way? (*T adds Roman numbers on the top*)… Does this help to understand that they are consecutive?

30 Voices: no.

The students, supported by D who suggests to substitute specific values to n, are able to highlight that the representations proposed by Alb are characterised by the fact that the first and the third numbers are the same, therefore they do not represent three consecutive numbers. Moreover, the expression n1+n2+n3, proposed by Edel's group, are too general because they only represent the sum of three numbers.

42 Pir: I can write n and after I change the letter. Different letters.

43 D: But it is the same objection I did for n1, n2, n3… how can I know they are consecutive numbers?

45 Vic: We can write… in the first case $n\cdot 2$, after $n\cdot 2+1$, after $n\cdot 2+3$.

46 T: Plus?

47 Vic: +2.

T writes on the whiteboard the expression $n\cdot 2+ n\cdot 2+1+ n\cdot 2+2$.

48 D: Did you understand what is it? Vic, could you explain it?

49 Vic: $n\cdot 2$ is an even number, $n\cdot 2+1$ is the consecutive…

50 D: Let's try and give some numeric values.

Vic proposes to substitute n=3 in the expression. Other students declare that Vic's expression is right.

55 D: Is this ok? This is a way of writing three generic consecutive numbers, isn't it?

Other students agree with Vic's proposal. D asks whether 5+6+7 can be written in that way and Bes proposes to change the representation into n+n+1+n+2.

64 T: Here, let's check whether we can write also an odd number.

65 Bes: n=5, you can do n=5, n+1=6 and n+2=7.

66 Vic: Or you can modify the above case, the second case says that…

D promotes a comparison between Vic's idea of representing two cases and Bes' idea of creating a more general representation and asks to the class whether it is necessary, to the aim of proving the divisibility by 3, to distinguish the two cases.

68 Voices: No.

69 D: Then, we can write only one, that will be for instance n+n+1+n+2. By now what did we do? We just represented the sum of three consecutive numbers… By now we just wrote the sum of three consecutive numbers. What do we do with that writing? Now we can go on and write $n/3$ or something similar, but… I let you think in which way, using this writing, we can go on with the justification.

70 T: Why does writing it in this way is useful for us?

71 Vic: Because modifying we would get number + number + number +1+2 and then…

T writes at the whiteboard

72 Vic: Summing up we would get number +number + number +3.

73 D: And $n+n+n$, how can we write it?

74 Voices: $n\cdot 3$.

75 D: And at this point do I see that is a number divisible by 3?

76 Voices: yes.

If we focus on D's interventions, we can observe that, from the very beginning of the discussion (4), she often poses herself at a meta-level, acting as an *activator of reflective attitudes,* bringing to the fore the *teleological* dimension. Specifically, in line 4, D wants to elicit the aim of writing the algebraic expression (communicating or proving) because her objective is to intervene at two different levels: at *epistemic* level, enabling the students to realise that the representations are not correct; at *teleological* level, enabling them to highlight that the algebraic representation should not contain also the "resulting property" (divisibility by 3), that should be derived from the transformation of the algebraic expression "sum of three consecutive numbers". When she asks to the students to compare Edel's group's approach with the approach analysed previously (6), she also acts as a *reflective guide*, fostering the comparison between two different ways of facing the activity. This role is activated by D also when she asks the students to compare Alb's proposal to Edel's (19).

During the discussion, D often acts also as an *activator of interpretative processes*, trying to support the students in highlighting the meaning of the algebraic representations they propose (14, 24, 43, 55). At the same time, D acts as a *reflective guide* and as an *activator of reflective attitudes* because her aim is to make the students catch if the different representations are really correct or not. In this part of the discussion, therefore, D focuses on the *epistemic* aspects, disentangling them with the *communicative* and *teleologic* ones: the algebraic representation must be correct, not only easy to understand, and "transformable". The effectiveness of this approach is evident when Vic is able to highlight a problem connected to the expressions proposed by Alb (25), to make the meaning of these expressions explicit (27) and to propose a possible modification of these expressions to represent consecutive numbers (45). D acts as a *reflective guide* also in helping Vic express the meaning of his proposal to the classmates (48, 50).

The influence of D's approach on T's activation of roles that should be played is evident when, instead of commenting on Alb's proposal of distinguishing the three numbers simply writing "first, second and third" (28), T re-launches this suggestion to the whole class (29), acting as an *activator of* both *reflective attitudes* and *interpretative processes* with the aim of making the students identify the problem. Starting from this moment, it is possible to highlight what we call "*achieved harmony*" between T's and D's interventions, that is an evidence of T's intention of supporting D's approach through her interventions. When, for example, Bes, referring to Vic's observations, correctly suggests to write a more general expression that really represents the sum of three generic consecutive numbers, T supports Bes in checking the correctness of her algebraic expression and in explaining the effectiveness of her proposal (60-62-64).

After having acted again as a *reflective guide*, making the meaning of Vic's and Bes' suggestions more explicit (67), D shifts students' attention on the effectiveness of the last expression (n+n+1+n+2) in supporting the construction of a mathematical justification of the fact that this sum is always divisible by 3 (69). In particular, focusing on this objective, D is acting as an *activator of anticipating thoughts* because she wants the students to transform this expression with the aim of highlighting the observed property. Here again we can observe an *achieved harmony* between T's and D's interventions, because T acts to make the teleological level arise, re-launching D's question to the class (70). In this way she enables the students to highlight how to transform the expression n+n+1+n+2 to show that it always represents a number that is divisible by 3.

COMMENTS AND CONCLUSIONS

Our working hypothesis was that the didactician, by her interventions during class discussions, may help the teacher carry out efficient ways to promote the student's rational behaviour in the use of algebra as a thinking tool. To test this hypothesis, we analysed a teaching episode, showing that the teacher, while working with the didactician acting as a $M_{AE}AB$ for the students, gradually activated specific roles in tune with the $M_{AE}AB$ construct. In particular, we introduced the idea of *"growing harmony"* to indicate those moments when the teacher starts proposing interventions, attitudes and behaviours in tune with the didactician's approach. In our opinion this "*growing harmony"* could represent an indicator of a deeper teacher's awareness about the ways in which she should behave to foster students' aware and effective use of algebraic language as a thinking tool.

In order to test this hypothesis, we will compare this discussion with other subsequent discussions carried out by the didactician and the teacher and by other teachers involved in the project. Moreover, we will interview the teachers to collect their narratives about their professional development path and, in tune with the methodology proposed by Cusi and Malara (2016), we will make the teachers refer to specific theoretical lenses (in particular the $M\text{-}_{AE}AB$ construct and Habermas' levels of rationality) in their a-posteriori reflections on the written transcripts of class discussions. In this way, it will be possible to highlight the teacher's growing awareness about both the meaning of the researcher's interventions and the crucial roles that should be played.

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