Designing a web-based professional development toolkit for supporting the use of dynamic technology in lower secondary mathematics

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Cornerstone Maths was designed to support wide-scale student learning of key mathematical concepts using dynamic digital technologies. Moving the project to scale (over 150 schools) has necessitated a rethinking of the design of the professional development component to provide more appropriate support for 'within school' implementation and for scaling among all the teachers of mathematics in a school. We report the outcomes of the first phases of design research through which we have used our empirical research to inform the design of a web-based 'Cornerstone Maths Professional Development Toolkit' created to achieve the afore-mentioned goals, describe some preliminary findings in terms of its use by different teachers and set out our plans for the future.

Keywords: lower secondary mathematics, dynamic mathematical technology, professional development, landmark activities, mathematical pedagogic practices

INTRODUCTION

The context of a longitudinal project in England, *Cornerstone Maths*, which aims to support wide-scale student access to dynamic mathematical technologies to enhance mathematical understanding of 'hard-to-teach' topics in lower secondary mathematics, has necessitated a highly connected approach to the three important themes of the conference: mathematics teaching; resources; and professional development. The evolution of these three elements has been central to the design research methodology that led to the definition of the *curriculum activity system* that comprised: dynamic web-based software; student workbook and teacher guide; and teacher professional development (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Vahey, Knudsen, Rafanan, & Lara-Meloy, 2013). The outcomes of this earlier work have been widely reported (Clark-Wilson, Hoyles, Noss, Vahey, & Roschelle, 2015; Hoyles, Noss, Vahey, & Roschelle, 2013). Although there is evidence of successful scaling of the Cornerstone Maths teaching approaches in particular school settings, as our focus has shifted to try to understand and theorise on the 'products and processes of scaling' (Clark-Wilson, Hoyles, & Noss, 2015), our research lens is now trained firmly on the nature of the specific mathematical knowledge for teaching that underpins classroom implementations of Cornerstone Maths in ways that retain fidelity to the original design principles. Furthermore, our extensive classroom observations are enabling an articulation of the nature of teachers' mathematical knowledge for teaching (MKT) and associated mathematical pedagogic practices (MPP) of teachers as they develop both confidence and competences in their classroom uses of the technology with their students.

THEORETICAL FOUNDATIONS

In keeping with the three themes of the conference, this section summarises the theoretical foundations of the current research and its focus on a need to better understand the important components of teachers' MKT and associated MPP when designing professional development to support teachers to work with dynamic mathematical technologies in classrooms.

Mathematics teaching with technology

The design principles of the Cornerstone Maths curricular activity system are deeply rooted in a number of seminal research projects, through which the efficacy of the teaching approaches with technology were explored and established. The three curriculum units address the following topics:

- **Unit 1 Linear functions**. Drawing on the seminal research of Jim Kaput (Kaput, 1987; Tatar et al., 2008), the unit addresses the following key mathematical ideas: coordinating algebraic, graphical, and tabular representations of linear functions; y = mx + c as a model of constant velocity motion; the meaning of m and c in the motion context; and velocity as speed with direction.
- **Unit 2 Geometric similarity**. The use of sliders to explore multiple instantiations of geometric figures within dynamic environments (Hollebrands, Laborde, & Sträßer, 2008) is central to the design of the unit, which addresses: identifying variants and invariants in shapes that are mathematically similar, including identification of the scale factor of enlargements and the particular conditions for congruency; and recognising the important one-to-one geometric correspondence of sides and vertices in mathematically similar polygons.
- **Unit 3 Algebraic patterns and expressions**. The ESRC/EPRSC-funded MiGen project[1] developed the microworld, 'eXpresser' and researched its impact on students' understanding of algebraic variable and generalisation within the context of geometric patterns (Mavrikis, Noss, Hoyles, & Geraniou, 2013). This software and tasks informed the design of the Cornerstone Maths unit, which addresses: recognising the geometric structure of algebraic patterns (seeing the general in the particular); naming and linking variables; and modelling algebraic equivalence through the different ways of seeing a pattern.

Each unit of work includes between 2-4 weeks of curriculum work, which schools implement as 'replacement units' within their localised 'scheme of learning'.

Conceptualising mathematics teachers' mathematical knowledge and practices with technology – the landmark activity

In our work, we use Thomas and Palmer's 'Pedagogical Technical Knowledge' (PTK), as a frame that incorporates 'the principles, conventions and techniques required to teach mathematics through the technology' (Thomas & Palmer, 2014, p. 75). PTK combines teacher factors such as instrumental genesis (Artigue, 2002; Guin & Trouche, 1999; Verillon & Rabardel, 1995), mathematical knowledge for teaching (MKT, Ball, Hill, & Bass, 2005) and teachers' orientations and goals (Schoenfeld, 2008) in a model as shown in Figure 1.

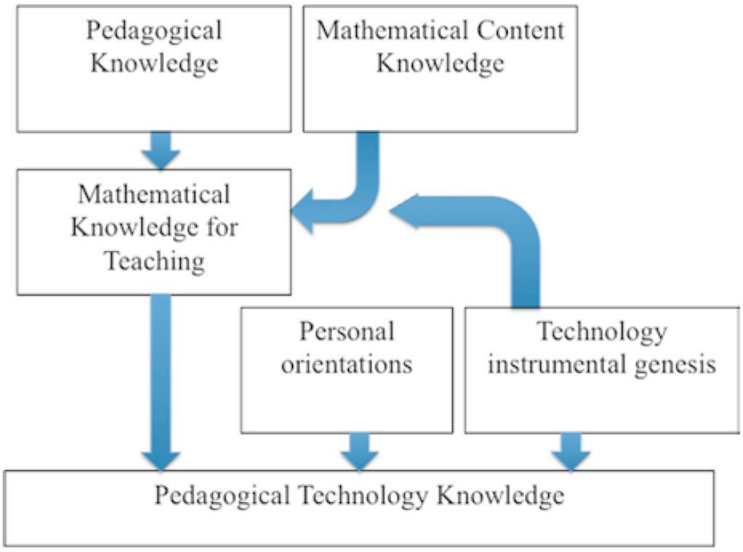

Figure 1: A model for the framework of PTK (Thomas & Palmer, 2014, p. 76)

Important to us was that PTK acknowledges teachers' personal orientations and the epistemic value of the tool, two elements that are absent in alternative frameworks. For example, in TPACK (Koehler & Mishra, 2009), teachers' technological knowledge is conceptualised as separate from the learning of the subject, drawing from the 'Fluency of Information Technology' as its theoretical base. Clearly teachers do need skills other than those around mathematical learning (for example classroom management with technology and some basic appreciation of the laptop, tablet or accessing the web) but this is not our main concern.

However, as we began to apply Thomas and Palmer's PTK in our work, we became interested in how it might be used to make sense of, and characterise teachers' mathematical pedagogic practices (MPP) with technology in classrooms. For this we looked to the work of Selling, Garcia and Ball (2016) who, in research to develop a framework for the design of items to assess teachers' MKT, have defined the 'mathematical work of teaching framework' (MWT) as a set of 'actions with and on objects' that relate to: 'mathematical representations; structure and explanations (including justifications and reasoning); and explanations (includes justifications & reasoning)' (ibid, p. 87).

We adapted this framework to take account of the use of digital tools and to devise the following set of pedagogic practices that we could use to both analyse classroom observation data and to inform the design principles for the Cornerstone Maths Professional Development (PD) Toolkit (See Table 1).

	Mathematical Pedagogic Practices with Technology
Explanations (includes justifications & reasoning)	**Comparing explanations that involve hypothesised or real actions as expressed with the digital tools** to determine which is more/most valid, generalisable, or complete explanation. **Critiquing explanations that involve hypothesised or real actions as expressed with the digital tools to improve them** with respect to completeness, validity, or generalisability.
Mathematical structure	**Determining, analysing, or posing problems as expressed with the digital tools** with the same (or different) mathematical structure. **Analysing structure in students' technological work** by determining which strategies or ideas are most closely connected with respect to mathematical structure. **Matching investigations with structure** as expressed by the digital tools.
Representations	**Connecting or matching representations** as expressed with the digital tools. **Analyzing representations** by identifying correct or misleading representations in a text, talk, written and technological work. **Selecting, creating, or evaluating different representations** as expressed by the digital tools. **Verbalising the meaning of representations** as expressed by digital tools and how they are connected to key ideas.

Table 1: Mathematical Pedagogic Practices for teaching mathematics with dynamic technology.

Scaling teachers' access to professional development

Our earlier work used research findings from the scaling of Cornerstone Maths in hundreds of English mathematics classrooms to articulate the 'processes and products of scaling' (See Table 2). By products, we mean the quantifiable measures that indicate the 'spread' of the Cornerstone Maths innovation across and within schools. The 'processes', or the means through which this spread is achieved, are both contextually and culturally located, with each process interpreted differently depending on the prevailing mathematical culture in classrooms and associated institutional factors.

Theme	Products	Processes
Geographical reach	a) Number of schools involved	a) Development of web-based curriculum activity system. b) Development of teacher community.
	b) Number of local hubs involved	c) Development and maintenance of regional hub-based offer of professional support. d) Development of school clusters, supported by project team leading to development of local hubs with local Cornerstone Maths project lead.
School buy-in	c) Improved student attainment	e) School-devised methods to evaluate students' outcomes
	d) Number of whole departments involved	f) Development of school-based PD. g) Support to embed Cornerstone Maths within local of schemes of work.
	e) Wider use of the materials	h) Teacher use of the materials beyond their original project commitment. (e.g. with older classes or revision classes).
Penetration in mathematics department	f) Number of participating teachers in each school	i) Development of a lead practitioner (who may be the subject leader). j) Development of peer-support for participating teachers.

Table 2: The products and processes of scaling Cornerstone Maths in hundreds of classrooms in England

Previous phases of Cornerstone Maths research involved PD that was face-to-face and online (asynchronous/synchronous) – focusing on Processes (a), (b) and (c) to achieve impacts related to Products (a) and (c) (Clark-Wilson & Hoyles, 2015; Clark-Wilson, Hoyles, & Noss, 2015).

However, we had research data from one school that evidenced that it had accomplished Processes (f), (g) and (h) to achieve impacts in relation to Products (d) and (e). Consequently, our attention turned to the design of a PD Toolkit that could directly support schools with some experience of Cornerstone Maths to develop their own collaborative, school-based PD to enrol other mathematics colleagues for within-school scaling. Furthermore, the design of the PD Toolkit was informed by prior research into teacher professional development in England that highlighted more effective practices thus:

> One successful approach involves **collaborative communities of practice** of teachers working to enquire into their professional practice. Such communities are often kick-started and sustained **by outside expertise**, provided by maybe a 'trainer' or a university educator. The most successful professional development **pays attention to the development of the subject (mathematics or science), itself** and particularly student learning. (de Geest, Back, Hirst, & Joubert, 2009, p. 38)

In our case the 'outside' expertise was not to come from an outsider to the department, but would be provided by a member of the department, a 'Cornerstone Maths champion', who had already participated in Cornerstone Maths PD and, most importantly, taught and evaluated Cornerstone Maths 'landmark activities' with positive outcomes[2]. Thus the toolkit has 'authenticity' in that it supports their co-planning and provides links to their everyday practice, with opportunities to reflect on students' work and classroom activities (See the ZDM Special Issue on 'Evidence-based CPD: Scaling up sustainable interventions', Roesken-Winter, Hoyles, & Blömeke, 2015).

Critical to the design of the Cornerstone Maths Toolkit is the assumption that Cornerstone Maths teachers will be self-motivated to select from its resources to use with colleagues in their own departments. Thus the development of the toolkit is the object of iterative and collaborative design research to address the research question, what (digital) professional development content, activities and structures can best support school-based PD concerning Cornerstone Maths?

METHODOLOGY

The Cornerstone Maths PD Toolkit is a set of diverse web-based resources for secondary mathematics departments to support school-based PD leading to embedding Cornerstone Maths units within the school's localised schemes of work. Our focus for this paper is the design research undertaken to produce the toolkit: the design principles and first description of the toolkit. We focus on the Cornerstone Maths Unit 3 on algebraic patterns and expressions.

Our design research methodology involves the following phases:

- Systematic analysis of questionnaire data from all Cornerstone Maths teachers that asked them to outline their current (and anticipated future) PD needs (n= 127).
- A review of other PD toolkits and their design (e.g. mascil[3], FaSMEd[4], EdUmatics[5]).
- Interviews with self-nominating Cornerstone Maths champions (n=9) during which they critiqued and enhanced early PD toolkit designs.
- A case study of one school that had successfully implemented all three Cornerstone Maths units in its localised schemes of work.

- Observations of teachers involved in collaborative PD using Cornerstone Maths Toolkit resources.

FINDINGS

The analysis of teacher questionnaires indicated some common 'PD needs', which included:

- Mathematical tasks that supported teachers to reflect on appropriate mathematical content and progression for each of the curriculum topics (developing both mathematical and pedagogical aspects of MKT).
- Short video clips and guidance materials that introduce and support teachers' instrumental geneses, which includes consideration of how teachers can, in turn support and develop *students*' instrumental geneses.
- Exemplar students' digital and paper/pencil productions embedded within professional tasks for teachers.

To date we have completed the first three phases and have designed a draft toolkit that includes a set of resources from classroom practice and student responses derived from landmark activities that provoked 'transformational' discussion among teachers and students, alongside more general background to Cornerstone Maths and evidence of its effectiveness. A design challenge is to provide opportunities for participating teachers to develop the composite elements of PTK in engaging and meaningful ways. Whilst we chose not to make these elements explicit within the PD toolkit design, we have mapped the elements that are specific to each of the Cornerstone Maths topics, software and teaching materials. For example, the definition and linking of algebraic variables is fundamental knowledge to support teachers' technology instrumental geneses within Cornerstone Maths Unit 3.

From the work to date, we conjecture that that opportunities for (and perceptions of) collaborative, departmental-based PD vary from school to school due to a range of factors derived from different sources; the overall structure of the school, the experience of the subject leaders to name but two. We intend to probe these factors further in case studies in a schools selected according to their different profiles in order to tease out which resources teachers select from the Cornerstone Maths PD Toolkit to use in their departmental PD and why, and, ultimately the success or not of any in school scaling. We anticipate reporting some tentative findings at the ERME conference in October 2016.

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NOTES

1. The MiGen project was funded by the ESRC/EPSRC Teaching and Learning Research Programme (Technology Enhanced Learning; Award No: RES-139-25-0381).

2. We use our own construct of 'landmark activities', which as those which indicate a rethinking of the mathematics or an extension of previously held ideas – the 'aha' moments that show surprise – and provide evidence of students' developing appreciation of the underlying concept (This construct is described in more detail in Clark-Wilson, Hoyles, & Noss, 2015).

3. The mascil toolkit "designed to support the delivery or facilitation of professional development for teachers of mathematics and science" http://mascil.mathshell.org.uk/

4. The FASMEd toolkit to support teachers teachers in the use of formative assessment with low achieving students. https://toolkitfasmed.wordpress.com/

5. EdUmatics online PD resource for secondary mathematics to learn to use and integrate technology within their classrooms. http://www.edumatics.mathematik.uni-wuerzburg.de/en/

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